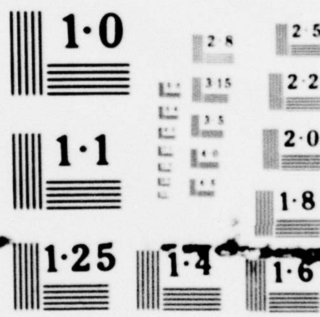
AD-A076 594 CONNECTICUT UNIV STORRS DEPT OF ELECTRICAL ENGINEERI--ETC F/G 17/7
MULTIPLE TARGET TRACKING WITH MEASUREMENTS OF UNCERTAIN ORIGIN.(U)
SEP 79 Y BAR-SHALOM , G D MARCUS N00014-78-C-0529
UNCLASSIFIED EECS-TR-79-14 NL

| OF |
AD-
A076594

END
DATE
FILMED
12-79
DDC



NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

(12)

AD A076594

The University of Connecticut
SCHOOL OF ENGINEERING
Storrs, Connecticut 06268

LEVEL II

DDC
NOV 14 1979
E

DDC FILE COPY

This document has been approved for public release and sale; its distribution is unlimited.

Department of

Electrical Engineering and Computer Science

79 10 16 108

12

DDC
RECEIVED
NOV 14 1979
E

MULTIPLE TARGET TRACKING WITH MEASUREMENTS OF UNCERTAIN ORIGIN

Y. Bar-Shalom and G. D. Marcus

Technical Report EECS-79-14

Annual Report

ONR Contract N00014-78-C-0529

September 1979

This document has been approved for public release and sale; its distribution is unlimited.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER EECS-79-14	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 Multiple Target Tracking with Measurements of Uncertain Origin.		5. TYPE OF REPORT & PERIOD COVERED 9 Annual Jun[redacted]78-Sep[redacted]79
		6. ~~PERFORMING ORG. REPORT NUMBER~~
7. AUTHOR(s) 10 Y./Bar-Shalom G. D./Marcus		8. CONTRACT OR GRANT NUMBER(s) 15 N00014-78-C-0529
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Connecticut, Department of EECS Box U-157, Storrs, Conn. 06268		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS ONR Code 431, Nav. Anal. Progr. Arlington, VA. 22217		12. ~~REPORT DATE~~ 11 Sep[redacted]79
		13. ~~NUMBER OF PAGES~~ 47
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 49		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

See last page

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same as 16

14 EECS-TR-79-14

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Tracking, Estimation, Multitarget, Sonar

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

See next page.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

410576

Abstract

The problem of estimating the state of a target in the presence of measurements of uncertain origin has received a great deal of attention recently. When this origin question cannot be resolved with certainty, one can only make probabilistic inferences as to which detection (and thus measurement) originated from the target of interest. The investigation reported here deals with a new class of problems characterized by the following: the correct measurement arrival (detection) times for a target of interest occur according to a stochastic process. Another stochastic process governs the arrival times of the false alarms. Thus detections occur one at a time and while some of them can be discarded as not having originated from the target, the remaining ones cannot be associated with certainty with the target. This problem is motivated by the fact that in some tracking problems detections from the target of interest occurs on an irregular basis. A procedure that associates probabilistically these measurements to the target is developed together with a corresponding estimator. The optimal estimator as well as a number of suboptimal algorithms that are real time implementable are presented together with simulation results. The simulations also indicate that the probabilistic data association filter with time-of-arrival information is significantly superior to the filter which uses only measurement location information for probabilistic data association.

The second part deals with the problem of joint probabilistic data association. In this case there are several targets and each measurement could have originated from one of these targets or none. The approach to this problem is to obtain joint probabilities for all the feasible associations of measurements to targets. An example is provided that illustrates that the results are substantially different from the case when marginal probabilities are obtained directly.

Accession For
NTIS GRA&I ✓
DDC TAB
Unannounced
Justification Per Ltr
on file
By
Distribution/
Availability Codes
Dist | Avail and/or special
A

0. Introduction

This report is organized in two parts. The first part deals with the case of a single target from which measurements are obtained at times governed by a stochastic process, (i.e. asynchronous measurements) with false measurements arriving at times governed by another stochastic process. The purpose of this study was to develop the methodology of incorporating the time-of-arrival information into the data association procedure. The results indicate both the feasibility as well as the significant performance improvements of the probabilistic data association method with time-of-arrival information. The second part deals with the probabilistic association of an arbitrary number of measurements to an arbitrary number of targets. The result, called Joint Probabilistic Data Association, accounts for all the possible origins of each measurement. References are presented following each part.

This study has been motivated by several practical problems encountered mainly in passive sonar tracking. The algorithms developed in the course of the study are feasible for real-time implementation and will allow more efficient use of the available information in tracking systems.

Part I

TRACKING WITH MEASUREMENTS OF UNCERTAIN ORIGIN AND RANDOM ARRIVAL TIMES

1. Introduction

The problem of estimating the state of a target in the presence of measurements of uncertain origin has received a great deal of attention recently. This attention was motivated by the fact that when tracking with certain sensors one can observe several detections whose origin might not have been the target of interest. When this origin question cannot be resolved with certainty, one can only make probabilistic inferences as to which detection (and thus measurement) originated from the target of interest. This problem arises mainly in passive sensors of acoustical or optical type and can be due to clutter or false alarms or interference from undesirable targets. In an environment where there are several targets as well as false alarms the ambiguities in the association of the measurements to the targets can become a rather severe problem.

The previous work related to this multi-target tracking problem, where there can be targets of interest, some of no interest and also false alarms (false targets), was recently surveyed in [B1]. The approaches discussed in the literature were either of the likelihood function or Bayesian type. They assumed that at predetermined times "snapshots" of the environment were obtained with several detections (and the corresponding measurements) that were to be associated into tracks.

The investigation reported here deals with a new class of problems characterized by the following: the correct measurement arrival (detection) times for a target of interest occur according to a stochastic process. Another stochastic process governs the arrival times of the false alarms. Thus detections occur one at a time and while some of them can be discarded as not having originated from the target, the remaining ones cannot be associated with certainty with the target. A procedure that associates probabilistically these measurements to the target is developed together with a corresponding estimator. This problem is motivated

by the fact that in some tracking problems contact with the target of interest occurs on an irregular basis. A related problem was studied by Nahi [N1] and Jaffer and Gupta [J1,J2] where they considered the measurements, obtained at fixed times, as being either a linear combination of the states plus noise or (zero-mean) noise alone. An estimation problem with measurements occuring at random times was considered in Snyder and Fishman [S2]. A combined estimation and control problem with some of the measurements occuring at random times was studied in Rhodes and Snyder [R1]. However, in these investigations there was no origin uncertainty about the observations.

The problem, formulated in Section 2, models the arrival times of the measurements as arbitrary renewal processes. The full memory Probability Data Association procedure, which carries all the origin hypotheses up to the current time, is developed for this model in Section 3, and the corresponding estimator in Section 4. Suboptimal procedures are presented in Section 5. The simplest of them, the Combined Single Hypothesis (CSH) filter, is only slightly more expensive to implement than the Standard Kalman Filter (SKF). The simulation results presented in Section 6 compare the SKF, CSH and a limited memory version of the PDA filter that retains the N most likely hypotheses. A significant improvement in performance has been observed for the CSH over the SKF. The additional improvement obtained over the CSH when carrying N=8 hypotheses was not so dramatic.

2. Formulation of the Problem

It is assumed that there is one target of interest, whose state x, an n-vector, evolves in continuous time according to the equation

$$\dot{x}(t) = A\, x(t) + B\, w(t) \tag{2.1}$$

where A and B are known matrices and w(t) is a zero-mean, white, Gaussian process noise with known covariance

$$E\, w(t)\, w'(\tau) = Q\, \delta(t-\tau) \tag{2.2}$$

A measurement corresponding to a detection that originated from the target being tracked is given by the equation

$$z(t_k) = H\, x(t_k) + v(t_k) \tag{2.3}$$

where t_k is the time at which the detection occurs* and v is a zero-mean white Gaussian noise sequence with covariance

$$E\, v(t_k)\, v'(t_j) = R_k\, \delta_{kj} \tag{2.4}$$

and independent of the process noise.

The set of measurements and the corresponding arrival times are denoted as

$$Z^k = \{z(t_i)\}_{i=1}^{k} \tag{2.5}$$

$$T^k = \{t_i\}_{i=1}^{k} \tag{2.6}$$

The minimum mean square error estimate of the state is then

$$\hat{x}(t_k|t_k) = E[x(t_k)|Z^k, T^k] \tag{2.7}$$

The arrival times of the measurements, both correct and false ones are modeled as renewal processes.

---

* The difference between "sensor time" and "target time" has been pointed out in [F1]. Here we do not make this distinction.

The pdf of the arrival time of the correct measurements is

$$p(t) = f_1(t-t_{\ell,1}) \qquad t > t_{\ell,1} \tag{2.8}$$

where $t_{\ell,1}$ is the time of the previous correct detection.

The pdf of the arrival time of the false measurements is

$$p(t) = f_0(t-t_{\ell,0}) \qquad t > t_{\ell,0} \tag{2.9}$$

where $t_{\ell,0}$ is the time of the last incorrect measurement prior to t. The symbols F_1 and F_0 will be used for the distributions corresponding to the densities in (2.8) and (2.9), respectively.

When a new measurement is received, say, at t_k, a validation region is set up around the predicted value of the measurement at t_k based upon past data. If the new measurement is outside this region it is discarded; otherwise it will be associated probabilistically to the track and used to update the target's estimate. This is discussed in detail later.

If the measurement is false, e.g., from clutter or false alarm, the spatial distribution of this measurement is assumed independent of past occurences and uniform within the surveillance region (with hypervolume V_s)

$$p\left[z(t_k) \mid \chi_{k,o}, Z^{k-1}, T^{k-1}\right] = p[z(t_k) \mid \chi_{k,o}] = 1/V_s \tag{2.10}$$

where we define as follows the events

$$\begin{aligned} \chi_{k,o} &\triangleq \{\text{detection at } t_k \text{ did not originate from the target}\} \\ \chi_{k,1} &\triangleq \{\text{detection at } t_k \text{ originated from the target}\} \end{aligned} \tag{2.11}$$

The information to be used in calculating the probability of each measure-

ment being correct (the probabilistic data association - PDA) is their arrival time and location. In order to use all the information, past history hypotheses

$$\chi^{k,i} = \{\chi^{k-1,q}, \chi_{k,j}\} \tag{2.12}$$

have to be stored and their probabilities evaluated. Each hypothesis is an assumption as to the origins of all the past measurements. This "full-memory" PDA is developed in the next section. It is somewhat similar to the "full scan back"* approach of Singer et al.[S1]. The main difference is that here one deals with random arrival times. Since the requirements of the full-memory PDA grow with time, "limited-memory" versions of it are also presented later.

*This terminology is radar oriented where several detections are obtained at each scan, i.e., at fixed times.

3. The Probabilistic Data Association for Measurements with Random Arrival Times - Full Memory

The conditional mean of the state (2.7) can be written using the total probability theorem as

$$\hat{x}(t_k|t_k) = \sum_i E[x(t_k)|\chi^{k,i}, Z^k, T^k]\, \beta^{k,i} \tag{3.1}$$

where

$$\beta^{k,i} \triangleq P\{\chi^{k,i}|Z^k, T^k\} \tag{3.2}$$

is the conditional probability that the i-th past history is correct. The evaluation of the probabilities (3.2) constitutes the PDA method. This method associates past sequences of measurements to the target in a probabilistic manner, hence its name [B2,S1].

Each estimate of the state vector conditioned upon a particular hypothesis on the r.h.s. of (3.1) is obtained by a standard linear (Kalman) filter.

The probabilities (3.2) will be evaluated next using Bayes' rule and the model of the measurements presented in the previous section.

$$\begin{aligned} P\{\chi^{k,i}|Z^k, T^k\} &= P\{\chi^{k-1,q}, \chi_{k,j}|Z^{k-1}, z(t_k), T^{k-1}, t_k\} \\ &= c^{-1} P\{\chi_{k,j}, z(t_k), t_k|\chi^{k-1,q}, Z^{k-1}, T^{k-1}\}\, P\{\chi^{k-1,q}|Z^{k-1}, T^{k-1}\} \\ &= c^{-1}\, p[z(t_k)|\chi^{k-1,q}, \chi_{k,j}, Z^{k-1}, T^{k-1}, t_k] \\ &\cdot P\{\chi_{k,j}, t_k|\chi^{k-1,q}, Z^{k-1}, T^{k-1}\}\, P\{\chi^{k-1,q}|Z^{k-1}, T^{k-1}\} \end{aligned} \tag{3.3}$$

where c is a normalizing constant. Each expression on the r.h.s. of (3.3) will be evaluated separately.

The first term is the pdf of the location of the last measurement and is either normal (for a correct measurement, i.e., $j=1$)* or uniform (for an incorrect detection, i.e., $j=0$)

*This follows from the linear-Gaussian model (2.1)-(2.4).

$$p[z(t_k)|\chi^{k-1,q}, \chi_{k,j}, Z^{k-1}, T^{k-1}, t_k]$$

$$= \begin{cases} \mathcal{N}[z(t_k); \hat{z}(t_k|t_{k-1}, \chi^{k-1,q}), S(t_k|t_{k-1}, \chi^{k-1,q})] & \text{if } j = 1 \\ V_s^{-1} & \text{if } j = 0 \end{cases} \quad (3.4)$$

where $\mathcal{N}[z; \hat{z}, S]$ is the normal density with variable z, mean $\hat{z}$ and covariance matrix S (which is the innovation covariance in this case conditioned on a particular hypothesis, to be discussed in more detail later); V_s is the hypervolume of the surveillance region.

The second term is the joint probability* of the event $\chi_{k,j}$ (that the detection at t_k is of type j) and the arrival time t_k of this detection conditioned on a past history hypothesis. Note that t_k is the realization of the random variable $\tau_{k-1,j}$ which is "the arrival time of the first detection of type j after t_{k-1}". Therefore this joint probability can be written as

$$P\{\chi_{k,j}, t_k | \chi^{k-1,q}, Z^{k-1}, T^{k-1}\}$$

$$= P\{\chi_{k,j} | \tau_{k-1,j} = t_k, \chi^{k-1,q}, Z^{k-1}, T^{k-1}\}$$

$$\cdot p_{\tau_{k-1,j}}[t_k | \chi^{k-1,q}, Z^{k-1}, T^{k-1}] \quad (3.5)$$

where the density above is subscripted to indicate the random variable it describes. Denote by $\theta_{k,j}$ the "last arrival time of a detection of type j prior to t_k" (in the entire surveillance region; this is yielded by the conditioning event $\chi^{k-1,q}$). Then one has

$$p_{\tau_{k-1,j}}[t_k | \chi^{k-1,q}, Z^{k-1}, T^{k-1}] = c_j^{-1} f_j(t_k - \theta_{k,j}) \quad (3.6)$$

---

*This is a mixed probability of an event and a continuous random variable. The notation used is the one for events, i.e., P.

where the notations of (2.8), (2.9) are used. The normalization constant c_j is needed because one has a truncated density: t_k must be larger than t_{k-1}. Thus

$$c_j = \int_{t_{k-1}}^{\infty} f_j(t_k - \theta_{k,j})\, dt_k = 1 - F_j(t_{k-1} - \theta_{k,j}) \tag{3.7}$$

Since event $\chi_{k,j}$ is equivalent to the event that the first detection from process j^* after t_{k-1} comes later than the one from process j, the first probability on the r.h.s. of (3.5) can be written as

$$P\{\chi_{k,j} \mid \tau_{k-1,j} = t_k, \chi^{k-1,q}, Z^{k-1}, T^{k-1}\}$$

$$= P\{\tau_{k-1,j^*} > \tau_{k-1,j} \mid \tau_{k-1,j} = t_k, \chi^{k-1,q}, Z^{k-1}, T^{k-1}\}$$

$$= \frac{1-F_{j^*}(t_k - \theta_{k,j^*})}{1-F_{j^*}(t_{k-1} - \theta_{k,j^*})} \tag{3.8}$$

where the denominator is again a normalization constant $c_j{}^*$ similar to (3.7). Note that, for any $\chi^{k-1,q}$ one of these two normalization constants will be unity.

The last term in (3.3) is available from the previous update. This completes the calculation of the a posteriori probabilities for each past history hypothesis.

4. The Estimator

Once the hypothesis probabilities have been obtained, the m.m.s.e. estimate of the target's state results from (3.1). Each hypothesis-conditioned estimate in (3.1) is obtained from a standard Kalman filter (which is the optimum estimator when conditioning on a hypothesis) together with an associated covariance matrix. Note that the conditional density of the target's state is a Gaussian mixture with $\beta^{k,i}$ the weightings. The covariance corresponding to the estimate (3.1) is [S1,B1]

$$
\begin{aligned}
P(t_k|t_k) = & \sum_i \beta^{k,i} P(t_k|t_k, \chi^{k,i}) \\
& + \sum_i \beta^{k,i} \hat{x}(t_k|t_k, \chi^{k,i}) \hat{x}'(t_k|t_k, \chi^{k,i}) \\
& - \hat{x}(t_k|t_k) \hat{x}'(t_k|t_k) \\
= & \sum_i \beta^{k,i} \Big\{ P(t_k|t_k, \chi^{k,i}) \\
& + [\hat{x}(t_k|t_k, \chi^{k,i}) - \hat{x}(t_k|t_k)] [\hat{x}(t_k|t_k, \chi^{k,i}) - \hat{x}(t_k|t_k)]' \Big\}
\end{aligned}
\tag{4.1}
$$

In order to reduce the computational requirements one can ignore in the PDA measurements that are "far" from the predicted measurement - i.e., one can use a validation region that eliminates the measurements which would be deemed by the PDA as having a very low probability of being correct. Since to set up an exact validation region based on the Gaussian mixture or a different one for each hypothesis is not practical, an elliptic region can be calculated assuming the state is approximately normally distributed with covariance as in (4.1). The innovation covariance at t_k is then

$$S(t_k|t_{k-1}) = H\,P(t_k|t_{k-1})\,H' + R_k \tag{4.2}$$

The state prediction covariance is obtained from

$$P(t_k|t_{k-1}) = \Phi(t_k,t_{k-1})\,P(t_{k-1}|t_{k-1})\,\Phi'(t_k|t_{k-1}) + \tilde{Q}(t_k,t_{k-1}) \tag{4.3}$$

where Φ is the transition matrix corresponding to A in (2.1); $P(t_{k-1}|t_{k-1})$ is given by the equivalent of (4.1) at t_{k-1}; the covariance $\tilde{Q}$ is the contribution of the continuous time process noise to the state prediction covariance

$$\tilde{Q}(t_k,t_{k-1}) = \int_{t_{k-1}}^{t_k} \Phi(t_k,\tau)\,B\,Q\,B'\,\Phi'(t_k,\tau)\,d\tau \tag{4.4}$$

The validation region at t_k is defined as

$$V_k = \left\{ z : [z - \hat{z}(t_k|t_{k-1})]'\,S^{-1}(t_k|t_{k-1})\,[z - \hat{z}(t_k|t_{k-1})] \le d \right\} \tag{4.5}$$

where d is taken from the chi-square tables and

$$\hat{z}(t_k|t_{k-1}) = H\,\Phi(t_k,t_{k-1})\,\hat{x}(t_{k-1}|t_{k-1}) \tag{4.6}$$

5. Limited Memory Procedures

Since the full-memory Probabilistic Data Association Filter described in Sections 3 and 4 has growing memory and computational requirements (2^N hypotheses can be generated after N detections, each of them requiring a Kalman filter) it is not feasible for implementation. This leads to the investigation of suboptimal algorithms.

One suboptimal tracking scheme, which eliminates the exponentially increasing memory requirements of the full memory filter can be obtained along the lines of the PDAF [B1,B2], which has also been called "zero-scan-back" filter [S1]. In this case instead of all the hypothesis-conditioned estimates (i.e. conditioned on a specific past history), a single estimate is generated and retained. Thus, unlike the full memory procedure, in this approach, to be called "combined single hypothesis" (CSH), one does not have to store track history hypothesis. In this procedure when a detection is obtained the probability of the corresponding measurement being correct is computed and the state estimate and its covariance are updated using this information.

The approximate sufficient statistic that replaces the past data in the PDA consists in this case of the following:

(i) the predicted state $\hat{x}(t_k|t_{k-1})$ and associated covariance matrix;

(ii) the arrival times $\theta_{k,j}$, $j=0,1$ of the latest detection of type j prior to t_k.

In the full memory approach these times are obtained from the history hypotheses. In the CSH procedure the arrival time of the last target-originated detection prior to time k will instead be determined from the probabilistic data association as follows. The last time the probabilistic data association calculated that the probability of a detection having originated indeed from the target was greater than the probability it was a false alarm will be declared

to be $\theta_{k,1}$. That is, the most recent time that a detection was more likely to have been associated with the target than with background clutter is considered to be the last occurrence of a target-originated detection:

$$\theta_{k,1} = t_{\ell_1} \quad \text{where} \quad \ell_1 = \max_{i<k} \{i : \beta_{i,1} > \beta_{i,0}\} \tag{5.1}$$

Similarly, $\theta_{k,0}$, the last occurrence prior to t_k of a false detection is taken as

$$\theta_{k,0} = t_{\ell_0} \quad \text{where} \quad \ell_0 = \max_{i<k} \{i : \beta_{i,0} > \beta_{i,1}\} \tag{5.2}$$

When a detection occurs two hypothesis-conditioned estimates are calculated. The conditioning is an origin hypothesis of only the current detection which yields

$$\hat{x}(t_k | t_k, \chi_{k,0}) = \hat{x}(t_k | t_{k-1}) \tag{5.3}$$

and $\hat{x}(t_k | t_k, \chi_{k,1})$ obtained from a standard Kalman filter.

The final state estimate is then a weighted sum of these two hypothesis-conditioned estimates. The weighted factors are the a posteriori probabilities that the origin hypothesis is correct.

$$\hat{x}(t_k | t_k) = \sum_{j=0}^{1} \hat{x}(t_k | t_k, \chi_{k,j}) \, \beta_{k,j} \tag{5.4}$$

where

$$\beta_{k,j} = P\{\chi_{k,j} | t_k, z(t_k), \hat{z}(t_k | t_{k-1}), \theta_{k,0}, \theta_{k,1}\} \qquad j=0.1. \tag{5.5}$$

Thus, the probabilistic data association of the CSH algorithm calculates the probability of an origin hypothesis of only the current detection being correct. In the full memory procedure the probability of a hypothesis as to the origin of each detection from initial to present time must be determined. The two probabilities of (5.4) are calculated using Bayes' rule as in Section 3.

This completes the description of the probabilistic data association for the CSH procedure. The covariance of the state estimate (5.4) must be determined. Equation (4.1) can be used. However, due to the nature of the CSH approach, the following simpler expression for the covariance of the unconditional estimate is obtained.

$$P(t_k|t_k) = \beta_{k,0}\, P(t_k|t_k,\chi_{k,0}) + \beta_{k,1}\, P(t_k|t_k,\chi_{k,i})$$
$$+ \beta_{k,0}\, \beta_{k,1}\, \mu_{k,1}\, \mu_{k,1}' \qquad (5.6)$$

where

$$\mu_{k,1} = x(t_k|t_k,\chi_{k,1}) - \hat{x}(t_k|t_{k-1}) \qquad (5.7)$$

Beyond this simplest approach there are a number of more complex filters to be discussed next.

A second suboptimal filter, which was implemented, is one in which a fixed maximum number N of hypotheses are retained. For example, once the number of hypotheses exceeds N, only the N most likely ones are retained and the others are discarded. The probabilities of the retained hypotheses are then renormalized such that they sum up to unity. Such a scheme is simple to implement and will give the optimum performance when the retained probability mass is near unity.

Other approaches can also be taken: one can keep a predetermined probability mass, e.g. 99%, regardless of the number of hypotheses to be retained or one can combine hypotheses that are similar by some relevant standard.

6. Simulation Results

The problem which was simulated was the tracking of a target with a four-dimensional state, consisting of position and velocity in two cartesian coordinates, driven by white noise.

Using the notations of eqs. (2.1) - (2.4), the system is characterized by the following state vector

$$x = [\ \xi\ \dot{\xi}\ \eta\ \dot{\eta}\ ]' \tag{6.1}$$

where ξ and η denote the cartesian coordinates and the system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.2}$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{6.3}$$

The process noise covariance matrix was taken as

$$Q = \begin{bmatrix} .01 & 0 \\ 0 & .01 \end{bmatrix} \tag{6.4}$$

and the measurement matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.5}$$

with measurement noise covariance

$$R = \begin{bmatrix} 9 \times 10^{-2} & 0 \\ 0 & 9 \times 10^{-2} \end{bmatrix} \tag{6.6}$$

The probability densities of the inter-arrival times of both the correct measurements as well as the false alarms were chosen from the gamma density family

$$f_1(t) = \frac{\lambda_T}{(m-1)!} (\lambda_T t)^{m-1} e^{-\lambda_T t} \qquad t > 0 \tag{6.7}$$

for the target - originated detections and for the false detections

$$f_o(t) = \frac{\lambda_F V_s}{(m-1)!} (\lambda_F V_s t)^{m-1} e^{-\lambda_F V_s t} \qquad t > 0 \tag{6.8}$$

where λ_T is in units $(\text{time})^{-1}$, λ_F is in units $(\text{time.volume})^{-1}$ and V_s is the size of the surveillance region. The average time between detections originating from the target is m/λ_T, from the false alarms is $m/(\lambda_F V_s)$. The following were the parameter choices:

$$m = 2 \tag{6.9}$$

$$\lambda_T = 1 \tag{6.10}$$

and λ_F assumed a number of values to simulate various degrees of environmental interference.

The validation region size was set such that the probability that a detection originating from the target falls in it was .999.

The threshold in (4.5) was set assuming a chi-square density, as

$$d = 13.8 \tag{6.11}$$

Three filters were implemented:

(i) a standard Kalman filter (SKF) which processed any detection lying within its validation region as if it were target originated;

(ii) the CSH filter

(iii) a fixed memory filter in which the N most likely tracks were retained.

The purpose of implementing the standard filter is to compare its performance against the two PDA filters.

The comparison was performed via a Monte Carlo simulation for various clutter densities. Each run lasted 30 units of time and an average was obtained over 100 runs. Measures used to compare the performance of the filters included the Cartesian norm of the position and velocity estimation error at time t_L of the last detection,

$$||\tilde{\mathbf{x}}_p|| = || H_p[\mathbf{x}(t_L) - \hat{\mathbf{x}}(t_L|t_L)]|| \tag{6.12}$$

$$||\tilde{\mathbf{x}}_v|| = || H_v[\mathbf{x}(t_L) - \hat{\mathbf{x}}(t_L|t_L)]|| \tag{6.13}$$

where

$$\mathbf{H}_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.14}$$

$$H_v = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.15}$$

The normalized position and velocity errors

$$\begin{aligned} \epsilon_p = {} & [x(t_L) - \hat{x}(t_L|t_L)]' H_p' \, [H_p \, P(t_L|t_L) \, H_p']^{-1} \\ & \cdot H_p[x(t_L) - \hat{x}(t_L|t_L)] \end{aligned} \tag{6.16}$$

and ϵ_v with a similar expression were also evaluated. This was done to check the consistency of the filter, i.e., whether its errors were compatible with the calculated covariances. The expected values of ϵ_p and ϵ_v are 2 because they have, theoretically, a chi square distribution with two degrees of freedom, assuming no lost tracks.

In addition, the number a times the filter "lost" the target was determined. A target was considered to be lost when more that 25% of the actual target-originated detections fell outside a filter's validation region.

The parameter of our study was the density of false alarms. The parameter entering into (6.8), $\lambda_F V_s$, was taken as 5, 10, 20 and 30 for $V_s = 1600$. To get an idea of the severity of these environments Table 6.1 shows the average number of false detections in the validation region of the standard filter.

$\lambda_F V_s$	5	10	20	30
Average Number of false detections	1.3	2.7	5.2	7.3
Expected number of correct detections	15	15	15	15

Table 6.1 Effective rate of false vs. true detections

The percentage of lost tracks is presented in Table 6.2. As it can be seen, for a relatively low rate of false alarms the SKF has already a significant probability of losing the track. The number of lost tracks for both the CSH and fixed memory filters is substantially lower than the SKF for all clutter densities.

$\lambda_F V_s$ / Filter	5	10	20	30
SKF	.17	.31	.64	.77
CSH	.02	.05	.09	.19
Fixed memory N = 8	.02	.04	.07	.11

Table 6.2 Probability of Losing the Target

Tables 6.3 and 6.4 present the position and velocity errors, respectively, for the three filters. For each clutter density, in both position and velocity, the SKF has the largest error Cartesian norm followed by the CSH, with the fixed memory filter having the lowest error norm in each case. In addition, the SKF is very "optimistic": the real time covariances are much too small. The performance of the standard filter is degraded and the covariance of the estimate does not increase to correspond to this situation. The normalized errors of both PDA filters are larger than the expected value of 2 (based on no lost tracks) which is due to the times these filters actually lost the target.

Filter \ $\lambda_F V_s$		5	10	20	30	40
SKF	$\|\|\tilde{x}_p\|\|$	9.17	1.05×10^1	1.97×10^1	3.61×10^1	2.81×10^1
	ϵ_p	4.20×10^3	1.12×10^3	2.99×10^3	1.04×10^4	8.48×10^3
CSH	$\|\|\tilde{x}_p\|\|$	5.21×10^{-1}	4.82×10^{-1}	1.17	6.00	3.07
	ϵ_p	1.45	1.50	2.35	2.41×10^1	9.50
Fixed memory with N=8 Hypotheses	$\|\|\tilde{x}_p\|\|$	5.21×10^{-1}	4.90×10^{-1}	5.42×10^{-1}	3.09	2.03
	ϵ_p	1.47	1.59	1.66	1.06×10^1	1.55×10^1

Table 6.3 Sample means of position errors.

Filter \ $\lambda_F V_s$		5	10	20	30	40
SKF	$\|\|\tilde{x}_v\|\|$	5.64×10^{-1}	5.40×10^{-1}	9.67×10^{-1}	1.60	1.19
	ε_v	2.96×10^{1}	1.03×10^{1}	3.82×10^{1}	1.06×10^{2}	5.38×10^{1}
CSH	$\|\|\tilde{x}_v\|\|$	2.25×10^{-1}	2.00×10^{-1}	2.94×10^{-1}	4.57×10^{-1}	3.03×10^{-1}
	ε_v	1.64	1.31	1.87	6.00	2.14
Fixed memory with N=8	$\|\|\tilde{x}_v\|\|$	2.21×10^{-1}	2.03×10^{-1}	2.34×10^{-1}	3.78×10^{-1}	2.90×10^{-1}
	ε_v	1.61	1.36	1.81	4.20	2.70

Table 6.4 Sample means of velocity errors.

The two PDA filters discussed above use both locations and arrival times to estimate the state of the system. To determine the effect of using arrival time information on tracking performance, a "combined single hypothesis with no time" filter (CSH-NT) was run incorporating only location information in the PDA. The comparison of tracking ability for the two CSH filters is displayed in Table 6.5.

$\lambda_F V_s$ / Filter	5	10	20	30
SKF	.17	.31	.64	.77
CSH-NT	.05	.18	.33	.52
CSH	.02	.05	.09	.19

Table 6.5 Probability of Losing the Target

These results indicate that significant improvement in tracking ability can be realized from use of arrival time information.

7. Conclusion

An algorithm has been developed to track a target where the origin of any detection is uncertain. In addition, the detections do not occur at fixed intervals, but instead the time intervals between detections are random. Since the optimal procedure requires exponentially increasing memory and computation, suboptimal versions are presented. A simulation was performed to compare the performance of these algorithms versus the standard filter.

The least expensive filter that uses a PDA procedure - the combined single hypothesis (CSH) algorithm - provides a substantial performance improvement over the standard filter. The fixed memory filter, which carries the N=8 most likely hypotheses, yielded a performance which is probably very close to optimum. It provided further improvement over the CSH, but at a substantial cost - it is 8 times more expensive than the CSH. The trade-off between performance and cost is a question which must be judged within the context of an individual problem. The deletion of the time of arrival information from the CSH filter resulted in significant performance deterioration. This indicates that the time of arrival information can be used successfully to obtain significant tracking performance improvement in this class of problems.

Acknowledgement

Stimulating discussions conducted with Dr. Edward P. Loane are gratefully acknowledged.

REFERENCES

[B1] Y. Bar-Shalom, "Tracking Methods in a Multitarget Environment," IEEE Trans. Auto. Control, Vol. AC-23, August 1978.

[B2] Y. Bar-Shalom, and E. Tse, "Tracking in a Cluttered Environment with Probabilistic Data Association", Proc. of the Fourth Symp. on Nonlinear Estimation, U.C. San Diego, Sept. 1973 and Automatica, Vol.11, pp. 451-460, Sept. 1975.

[F1] T.E. Fortmann and S. Baron, "Octopus, An Experimental Passive Sonar Tracking System" (SECRET), Proc. ONR Passive Tracking Conference, Monterey, Calif., May, 1977.

[J1] A.G. Jaffer and S.C. Gupta, "Recursive Bayesian Estimation with Uncertain Observation", IEEE Trans. on Information Theory, Vol. IT-17, pp. 614-616, Sept. 1971.

[J2] A.G. Jaffer and S.C. Gupta, "Optimal Sequential Estimation of Discrete Processes with Markov Interrupted Observations", IEEE Trans. on Automatic Control, Vol. AC-16, pp. 471-475, October 1971.

[N1] Nahi, N.E., "Optimal Recursive Estimation with Uncertain Observation", IEEE Trans. on Information Theory, Vol. IT-15, pp. 457-462, July 1969.

[P1] A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, Inc., 1965.

[R1] I.B. Rhodes and D.L. Snyder, "Estimation and Control of Performance for Space-Time Point-Process Observations", IEEE Trans. Auto. Control, Vol. AC-22, pp. 338-345, June 1977.

[S1] R. Singer, R. Sea and K. Housewright, "Derivation and Evaluation of Improved Tracking Filters for Use in Dense Multitarget Environments," 1973 Symp. on Information Theory, and IEEE Trans. Info. Theory, Vol. IT-20, pp. 423-432, July 1974.

[S2] D.L. Snyder and P.M. Fishman, "How to Track a Swarm of Flies by Observing their Flashes", IEEE Trans. Info. Theory, Vol. IT-21, pp. 692-695, Nov. 1975.

Part II

SIMULTANEOUS PROBABILISTIC DATA ASSOCIATION FOR SEVERAL TARGETS

1. Introduction

The problem of associating probabilistically measurements to a number of known targets (established tracks) was first considered in [B1]. The resulting algorithm, which was an extension of the results of [B2], used a diffuse prior for the false measurements. While this was shown to be equivalent in the single target case to a Poisson model (see [B3]) it was not adequate for the multiple target case. More recently, an algorithm was presented in [R1] that yielded probabilities of measurement histories being a track. While this latter algorithm is more general than all the previous ones, it has a number of limitations. Its feasibility of application for complex state models where the state is not completely observable from one sensor is an open question. The model used for "new targets", which consisted of an arrival process with strictly positive rate can lead to saturation of the surveillance region.

The algorithm to be presented in Section 2 deals with the association of an arbitrary number of measurements to an arbitrary number of established tracks (targets).

2. The joint PDA for targets with common detections

For the sake of illustration consider first the case with n=2 known targets.

Let the validation regions ("gates") of the two targets be

$$V_j = \{z : || z - \hat{z}_j || \leq \gamma\} \qquad j=1,2 \tag{1}$$

These regions are depicted in Fig. 1 with a total number of validated measurements m=4 including one which is common.

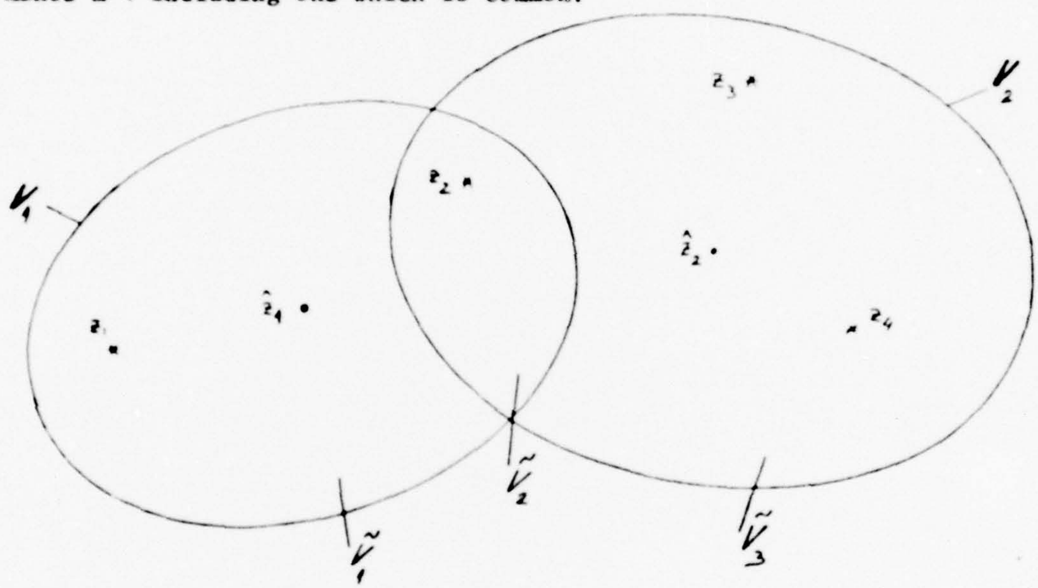


Fig. 1 Two validation regions with nonempty intersection

Next, consider the general case.

The validation matrix for m measurements and n targets is

$$\Omega = [\omega_{ij}] \qquad i=1,\ldots,m \qquad j=0,1,\ldots,n \tag{2}$$

with binary elements indicating whether measurement i has been validated for target j. Index j=0 stands for "clutter". The validation matrix corresponding to the above figure is (for n=2)

$$\Omega = \begin{array}{c|ccc|} & 0 & 1 & 2 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 \\ 4 & 1 & 0 & 1 \\ \hline \end{array} \tag{3}$$

The probabilities of joint events

$$\chi = \bigcap_{i=1}^{m} \chi_{ij_i} \tag{4}$$

where χ_{ij_i} means that measurement i originated from target j_i, will be computed.

To obtain the feasible events we shall use an indicator variable equivalent of χ_{ij}, namely

$$\hat{\omega}_{ij} = \begin{cases} 1 & \text{if } \chi_{ij} \text{ occurs} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Then to a joint event χ there corresponds a matrix

$$\hat{\Omega}(\chi) = [\hat{\omega}_{ij}(\chi)] \tag{6}$$

The conditions for an event to be feasible are

(i) only one unit from each row of Ω can be picked up for $\hat{\Omega}$

$$\sum_{j=0}^{n} \hat{\omega}_{ij}(\chi) = 1 \qquad i=1,\dots,m \tag{7}$$

(ii) at most one measurement can belong to each target

$$\delta_j(\chi) \triangleq \sum_{i=1}^{m} \hat{\omega}_{ij}(\chi) \le 1 \qquad j=1,\dots,n \tag{8}$$

The binary variable δ_j indicates whether target j was detected and its measurement validated in event χ.

Measurement i is associated with a target in event χ if

$$\tau_i(\chi) \triangleq \sum_{j=1}^{n} \hat{\omega}_{ij}(\chi) = 1 \tag{9}$$

An algorithm that obtains all the feasible matrices $\hat{\Omega}$ can be set up as follows

1. Scan through the (say, N) non-zero elements of Ω to set up all the possible binary numbers (2^N) from them
2. Eliminate all those that do not satisfy (7) and (8).

Using $\hat{\Omega}$ one can obtain the set

$$X_{ij} = \{ \chi : \hat{\omega}_{ij} (\chi) = 1 \} \tag{10}$$

needed to evaluate the marginal probability

$$\beta_{ij} = P \{x_{ij} | Z^k, \Omega\} = \sum_{\chi \in X_{ij}} P\{\chi | Z^k, \Omega\} \tag{11}$$

from the probabilities of the joint events χ.

The probabilities of the events χ we obtained from

$$P\{\chi | Z^k, \Omega\} = \frac{1}{c} p\{Z(k) | \chi, Z^{k-1}, \Omega\} P\{\Omega | \chi, Z^{k-1}\} P\{\chi | Z^{k-1}\} \tag{12}$$

where Z(k) the set of current observations, $Z^k = Z(k) \cup Z^{k-1}$ is the set of observations through k.

The first term on the r.h.s. of (12) is obtained next

$$p[Z(k) | \chi, Z^{k-1}, \Omega] = \prod_{i=1}^{m} p[z_i(k) | x_{ij}, Z^{k-1}, \Omega] \tag{13}$$

where

$$p[z_i(k) | x_{ij}, Z^{k-1}, \Omega] = \begin{cases} c_{jr_i}^{-1} \; f_j[z_i(k)] & j \neq 0 \\ \tilde{V}_{r_i}^{-1} & j = 0 \end{cases} \tag{14}$$

In the above $f_j(\cdot)$ is the density of the predicted measurement for target j(at the current time k); $c_{jr_i}^{-1} f_j$ is this density restricted to the region $\tilde{V}_{r_i}$ in which $z_i(k)$ lies. For example in Fig. 1 the three regions $\tilde{V}$ are illustrated. There is one such region for each different row of Ω and their total number is m_v. For the validation matrix (3) the indices r_i corresponding to each measurement in Fig. 1 are

i	1	2	3	4
r_i	1	2	3	3

The false measurements pdf in (14) is the inverse of the volume of the corresponding region

$$\tilde{V}_{r_i} = \mathrm{vol}\left\{\tilde{V}_{r_i}\right\} \tag{15}$$

The constraits c_{ir_i} are given by the expression

$$c_{jr_i} = \int_{\tilde{V}_{ri}} f_j[z_i(k)]\, dz_i(k) \tag{16}$$

which does not have to be calculated because it will cancel out as shown later.

The second term on the r.h.s. of (12) is

$$P\{\Omega|\chi,Z^{k-1}\} = P\left[\bigcap_{i=1}^{m}\{z_i(k)\epsilon\tilde{V}_{r_i}|x_{ij},Z^{k-1}\}\right]$$

$$= \left(\prod_{i:\tau_i(\chi)=1} P\{z_i(k)\epsilon\tilde{V}_{r_i}|x_{ij},Z^{k-1}\}\right)P\left[\bigcap_{i:\tau_i(\chi)=0}\{z_i(k)\epsilon\tilde{V}_{r_i}|x_{io}Z^{k-1}\}\right] \tag{17}$$

where the product is over the measurements associated with a target (since the corresponding events are independent); the second term contains the measurements not associated with any target.

Note that for a detection associated with a target the probability that the corresponding measurement falls in $\tilde{V}_{r_i}$, conditioned on χ_{ij} is

$$P\{z_i(k)\epsilon\tilde{V}_{r_i}|\chi_{ij},Z^{k-1}\} = \int_{\tilde{V}_{r_i}} f_j[z_i(k)]dz_i(k) \left[\int_{V_j} f_j[z_i(k)]dz_i(k)\right]^{-1}$$

$$\text{if } \tau_i(\chi) = 1 \tag{18}$$

The denominator in the above follows from the fact that if χ_{ij} is considered then z_i was validated for target j and thus its pdf is f_j divided by the probability that it fell in the gate

$$P_{G_j} \triangleq \int_{V_j} f_j(z)\ dz \tag{19}$$

Then using (16), Eq.(18) becomes

$$P\{z_i(k)\epsilon\tilde{V}_{r_i}|\chi_{ij},Z^{k-1}\} = c_{jr_i}\ P_{G_j}^{-1} \tag{20}$$

The last term in (17) can be rewritten as the product of probabilities that the number of false measurements ϕ in the nonoverlapping regions $\tilde{V}_{r_i}$, originating from a Poisson distribution with rate λ, are as specified by the event χ under consideration

$$P\left[\bigcap_{i:\tau_i(\chi)=o} \{z_i(k)\epsilon\tilde{V}_{r_i}|\chi_{io},Z^{k-1}\}\right]$$

$$= \prod_{\ell=1}^{m_v} P\{\phi(\tilde{V}_\ell)=\phi_\ell(\chi)\} = \prod_{\ell=1}^{m_v} e^{-\lambda\tilde{V}_\ell}(\lambda\tilde{V}_\ell)^{\phi_\ell}(\phi_\ell!)^{-1} \tag{21}$$

Combining (20) and (21) into (17) yields

$$P\{\Omega|\chi,Z^{k-1}\}=\prod_{i:\tau_i(\chi)=1} c_{jr_i}\ P_{G_j}^{-1} \prod_{\ell=1}^{m_v} e^{-\lambda\tilde{V}_\ell}\ (\lambda\tilde{V}_\ell)^{\phi_\ell}(\phi_\ell!)^{-1} \tag{22}$$

The last term from (12) is the prior (w.r.t. the current time k) probability of event χ. Note that (17) is the probability that

(i) the correct measurements implied by χ (i.e. given that they are detected and validated) fell into the corresponding nonoverlapping regions $\tilde{V}_{r_i}$

(ii) the incorrect measurements fell into the various regions $\tilde{V}_\ell$ as implied by χ: ϕ_ℓ in volume $\tilde{V}_\ell$, i.e., their number and apportionment among the regions.

In view of this the last term will reflect

(1) the probability of detection and validation of the correct measurements

(2) the probability that the measurements of the targets whose $\delta_j=0$ (no measurement associated with target j) were either not detected or fell outside the validation region.

It will be assumed that all joint events that have measurements from the same set of targets are a priori equiprobable.

Denote the binary vector that indicates which targets are assumed detected in an event χ as

$$\underline{\delta} = \underline{\delta}\,(\chi) \tag{23}$$

with components defined by (8). There are 2^n such different vectors where n is the number of targets. All joint events χ that have the same $\underline{\delta}$ are thus considered a priori equiprobable.

Let

$$q_{\underline{\delta}} = q(\underline{\delta},\Omega) \tag{24}$$

be the number of events χ with the same $\underline{\delta}$ as it follows from the matrix Ω (see Appendix A for the expression of this number and consistency check with [B2]). With this the prior probability of an event χ is

$$P\{\chi|Z^{k-1}\} = \frac{1}{q_{\underline{\delta}}} \prod_{j:\delta_j(\chi)=1} P_{D_j} P_{G_j} \prod_{j:\delta_j(\chi)=o} \left(1-P_{D_j} P_{G_j}\right) \tag{25}$$

Note that

$$\sum_{\chi} P\{\chi|Z^{k-1}\} = \sum_{\underline{\delta}} \prod_{j:\delta_j(\chi)=1} P_{D_j} P_{G_j} \prod_{j:\delta_j(\chi)=o} \left(1-P_{D_j} P_{G_j}\right)$$

$$= \prod_{j} \left(P_{D_j} P_{G_j} + 1-P_{D_j} P_{G_j}\right) = 1 \tag{26}$$

which indicates that the normalization constants in (25) are "natural". Combining (14), (22) and (25) into (12) yields

$$P\{\chi|Z^{k},\Omega\} = \frac{1}{c} \prod_{i:\tau_i(\chi)=1} c_{jr_i}^{-1} f_j[z_i(k)] \prod_{i:\tau_i(\chi)=o} \tilde{V}_{r_i}^{-1}$$

$$\cdot \prod_{i:\tau_i(\chi)=1} c_{jr_i} P_{G_j}^{-1} \prod_{\ell=1}^{m_v} e^{-\lambda\tilde{V}_\ell} (\lambda\tilde{V}_\ell)^{\phi_\ell} (\phi_\ell!)^{-1}$$

$$\cdot \frac{1}{q_{\underline{\delta}}} \prod_{j:\delta_j(\chi)=1} P_{D_j} P_{G_j} \prod_{j:\delta_j(\chi)=o} \left(1-P_{D_j} P_{G_j}\right)$$

$$= \frac{\lambda^{\phi}}{c' q_{\underline{\delta}}} \prod_{i:\tau_i(\chi)=1} f_j[z_i(k)] \prod_{\ell=1}^{m_v} (\phi_\ell!)^{-1}$$

$$\cdot \prod_{j:\delta_j(\chi)=1} P_{D_j} \prod_{j:\delta_j(\chi)=o} \left(1-P_{D_j} P_{G_j}\right) \tag{27}$$

where

$$\phi = \sum_{\ell=1}^{m_v} \phi_\ell (\chi) \tag{28}$$

and c_{ir_i}, $\tilde{V}_\ell$, P_{G_j} cancel immediately and the term

$$\prod_{\ell=1}^{m_v} e^{-\lambda \tilde{V}_\ell} = e^{-\lambda \sum_{\ell=1}^{m_v} \tilde{V}_\ell} = e^{-\lambda \tilde{V}} \tag{29}$$

appears regardless of which χ is under consideration and thus cancels between numerator and denominator in (27).

The probability of a joint event χ given in (27) was derived assuming that there are validation regions corresponding to each target. While the volumes of the validation regions cancelled in the final equation some factorials corresponding to the number of false detections subsumed in χ were left in the final expression. Also the normalization constant $q_{\underline{\delta}}$ defined in (24) appears in (27).

The derivation of the conditional probability of a joint event is done next assuming each validation region to coincide with the entire surveillance region. In this case the validation matrix (2) is made up of units everywhere. (However, for practical purposes one can still use validation regions to avoid considering events that will have negligible probability.) In this case there is no Ω in (12), which becomes

$$P\{\chi|Z^k\} = \frac{1}{c} p[Z(k)|\chi, Z^{k-1}] \, P\{\chi|Z^{k-1}\} \tag{30}$$

The first term above is

$$p[Z(k)|\chi, Z^{k-1}] = \prod_{i=1}^{m} p[z_i(k)|\chi_{ij}, Z^{k-1}] \tag{31}$$

where

$$p[z_i(k)|\chi_{ij}, Z^{k-1}] = \begin{cases} f_j[z_i(k)] \\ V^{-1} \end{cases} \tag{32}$$

with V being the volume of the entire surveillance region.

The "prior" probability of an event χ is

$$P\{\chi|Z^{k-1}\} = \frac{1}{\alpha(\chi)} \prod_{j:\delta_j(\chi)=1} P_{D_j} \prod_{j:\delta_j(\chi)=o} \left(1-P_{D_j}\right)$$

$$\cdot e^{-\lambda_F V} (\lambda_F V)^{\phi} (\phi!)^{-1} \tag{33}$$

where

$$\phi = \phi(\chi) \tag{34}$$

is the total number of false measurements in event χ and $\alpha(\chi)$ is the number of events χ with detections from the same set of targets. This is equal to the number of permutations of m (total number of detections) takes as $m-\phi$ (number of correct detections)

$$\alpha(\chi) = P^m_{m-\phi} = \binom{m}{m-\phi}(m-\phi)! = \frac{m!}{\phi!} \tag{35}$$

Inserting (31)-(35) into (30) yields

$$P\{\chi|Z^k\} = \frac{1}{c} \prod_{i:\tau_i(\chi)=1} f_i[z_i(k)]\ V^{-\phi}$$

$$\cdot \frac{\phi!}{m!} \prod_{j:\delta_j(\chi)=1} P_{D_j} \prod_{j:\delta_j(\chi)=o} \left(1-P_{D_j}\right)$$

$$\cdot e^{-\lambda_F V} (\lambda_F V)^{\phi} (\phi!)^{-1} \tag{36}$$

which becomes after cancellations

$$P\{\chi|Z^k\} = \frac{\lambda_F^{\phi(\chi)}}{c} \prod_{i:\tau_i(\chi)=1} f_j[z_i(k)] \prod_{j:\delta_j(\chi)=1} P_{D_j} \prod_{j:\ \delta_j(\chi)=o} \left(1-P_{D_j}\right) \tag{30}$$

which is consistent with the expression of [R1].

3. <u>An Example</u>

Consider two targets with a one-dimensional measurement with predicted locations for targets 1 and 2 as

$$\hat{z}_1 = 0, \ \hat{z}_2 = 3$$

and variance of the prediction S=1. The following three measurements were to be associated with the two targets

$$z_a = -1.5, \ z_b = 1.5, \ z_o = 4.5$$

This is depicted below. Only z_b was validated for both targets.

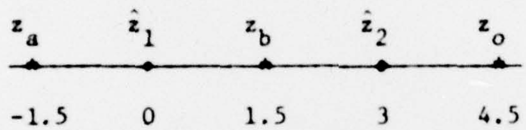


Table 1 presents the probabilities of all the joint events, assuming $P_D = 0.8$, for various densities λ for the false alarms. Table 2 shows the corresponding marginal probabilities.

Note that in the absence of target 2, the events $\chi_{a,1}$ and $\chi_{b,1}$ would have been equiprobable. Since z_b can also belong to target 2, the joint PDA yields a drastically different result. This illustrates the need to account for all the possible origins of each measurement.

a	b	c	λ=.1	λ=.05	λ=.02
0	0	0	.0097	.0027	.0004
1	0	0	.0506	.0284	.0122
0	1	0	"	"	"
0	2	0	"	"	"
0	0	2	"	"	"
1	2	0	.2625	.2945	.3168
1	0	2	"	"	"
0	1	2	"	"	"

Table 1. Joint probabilities for all the feasible associations for various false alarm densities.

χ	λ=.1	λ=.05	λ=.02
$\chi_{a,1}$	.5756	.6174	.6458
$\chi_{b,1}$	.3131	.3229	.3290
$\chi_{b,2}$	.3131	.3229	.3290
$\chi_{c,2}$	.5756	.6174	.6458

Table 2. Marginal probabilities for all the feasible associations.

4. Conclusion

Reid [R1] derived his expressions for the joint probabilities assuming

(i) The target detection events are identically distributed (same P_D) and independent

(ii) A number of new targets are detected according to a Poisson distribution (this implies a net inflow into the surveillance region, i.e., monotomically increasing number of targets)

(iii) Initialization of new targets from one measurement is possible

The JPDA approach described above is

1. More general than Reid's because the target detection events are not identically distributed
2. Less general because it assumes no new targets
3. More realistic because both assumptions (ii) and (iii) above do not seem to fit many practical situations, in particular the passive sonar tracking.

Appendix A

In the particular case of n=1 target the computation of the priors is done as follows. For χ_i, i=1,...,m ("the i-the validated measurement is correct") one has δ=1 ("target detected") and from (24)

$$q_1 = m \tag{A.1}$$

From (25) it follows that

$$P\{\chi_i | Z^{k-1}\} = \frac{1}{m} P_D P_G \tag{A.2}$$

For i=o one has δ=o and q_o=1, thus

$$P\{\chi_o | Z^{k-1}\} = 1-P_D P_G \tag{A.3}$$

Computation of the posterior probabilities is done next. Note that in (14) f is the nontruncated density of the predicted measurement. In the survey paper [B3] f was used as the truncated density, i.e., it included the factor P_G^{-1}. From (27) one has for i≠o (in which case ϕ=m-1, q_δ=m)

$$\beta_i = P\{\chi_i | Z^k, m_k = m\} = \frac{\lambda^{m-1}}{c'm} f(z_i) \frac{1}{(m-1)!} P_D \tag{A.4}$$

for i=o

$$\beta_o = P\{\chi_o | Z^k, m_k = m\} = \frac{\lambda^m}{c'} \frac{1}{m!} (1-P_D P_G) \tag{A.5}$$

The normalization constant c' is

$$c' = \frac{\lambda^{m-1}}{m!} P_D \sum_{i=1}^{m} f(z_i) + \frac{\lambda^m}{m!} (1-P_D P_G) \tag{A.6}$$

Then, for i=1,...,m

$$\beta_i = \frac{\frac{\lambda^{m-1}}{m!} P_D f(z_i)}{\frac{\lambda^m}{m!} (1-P_D P_G) + \frac{\lambda^{m-1}}{m!} P_D \sum_{i=1}^{m} f(z_i)} = \frac{f(z_i)}{b_o + \sum_{i=1}^{m} f(z_i)} \tag{A.7}$$

where

$$b_o = \lambda \frac{1-P_D P_G}{P_D} \tag{A.8}$$

and

$$\beta_o = \frac{b_o}{b_o + \sum_{i=1}^{m} f(z_i)} \tag{A.9}$$

which matches the earlier results of [B2].

Consider n=2 targets with the following number of validated measurements

m_{ii} - for target i only i=1,2

m_{ij} - for targets i and j (in the intersection of their validation regions)

Then the number of events for different $\underline{\delta} = [\delta_1, \delta_2]$ is

$$q(\delta_1=1, \delta_2=1) = m_{11} m_{22} + m_{11} m_{12} + m_{12} m_{22}$$

$$q(\delta_1=o, \delta_2=1) = m_{12} + m_{22}$$

$$q(\delta_1=1, \delta_2=o) = m_{11} + m_{12}$$

$$q(\delta_1=o, \delta_2=o) = 1$$

REFERENCES

[B1] Y. Bar-Shalom, "Extension of the Probabilistic Data Association Filter to Multitarget Environment," Proc. Fifth Symposium on Nonlinear Estimation, San Diego, CA, Sept. 1974.

[B2] Y. Bar-Shalom, and E. Tse, "Tracking in a Cluttered Environment with Probabilistic Data Association", Proc. of the Fourth Symp. on Nonlinear Estimation, U.C. San Diego, Sept. 1973 and Automatica, Vol. 11, pp. 451-460, Sept. 1975.

[B3] Y. Bar-Shalom, "Tracking Methods in a Multitarget Environment," IEEE Trans. Auto. Control. Vol. AC-23, August 1978.

[R1] D. B. Reid, "An Algorithm for Tracking Multiple Targets", Proc. 1978 IEEE Conf. on Dec. & Control, San Diego, CA, Jan. 1979.

Distribution

Mr. James G. Smith, Scientific Officer (1)
Code 431
Office of Naval Research
Arlington, VA 22217

ONR Branch Office (1)
Bldg. 114, Sec. D.
666 Summer Street
Boston, MA 02210

Mr. Fred Walters, Contract Administrator (1)
ONR
715 Broadway, 5th Floor
New York, NY 10003

Naval Research Lab (6)
Code 2627
Washington DC 20375

ONR (6)
Code 102D61
Arlington, VA 22217

DDC (12)
Bldg. 5, Cameron Station
Alexandria, VA 22314

Dr. S. I. Chou (1)
Code 7131
Naval Ocean System Center
San Diego, CA 92152

Mr. Jerry Hill (1)
Code 352
Naval Underwater System Center
Newport, RI 02840

Dr. Charles Morefield (1)
VERAC
4901 Morena Blvd. #209
San Diego, CA 92117

Prof. Michael Athans (1)
MIT, Rm. 35-209
Cambridge, MA 02139

Prof. Byron Tapley (1)
Dept. of Aerospace Engr.
Univ. of Texas
Austin, TX. 78712

Dr. Vincent Gabriel (1)
Farrell Rd. Plant
Bldg. 1, Rm D6
General Electric
Syracuse, NY 13221

Dr. T. Fortmann
BBN
50 Moulton St.
Cambridge, MA 02138 (1)

Dr. Hugh A. Reeder
Tracor, Inc.
6500 Tracor Lane
Austin, TX 78721 (1)